<u>Helium vs. Air in Football</u>

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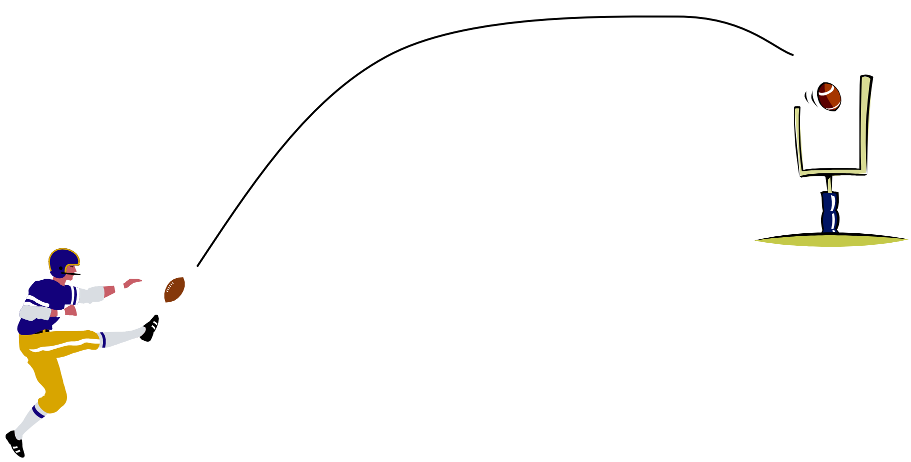
<u>Introduction</u>

The data was initially collected and recorded in the following manner. Two identical footballs were used in the experiment. One was filled with air and the other with helium. The experiment occurred at Ohio State University's athletic complex. The complex was outdoors, but the experiment occurred on a windless day so that the flight of the footballs would not be affected. A reliable and consistent kicker was selected for the experiment. The kicker was not informed of which football contained air and which contained helium. Each football was kicked 39 times, and the kicker changed footballs after each kick. The distance was then recorded for each kick from the placement of the kick to where the football eventually landed.

Ten of the 39 trials along with their corresponding values for the distances in yards were randomly selected without replacement. Figure A is a list of the 10 trials randomly selected and the corresponding values of distances.

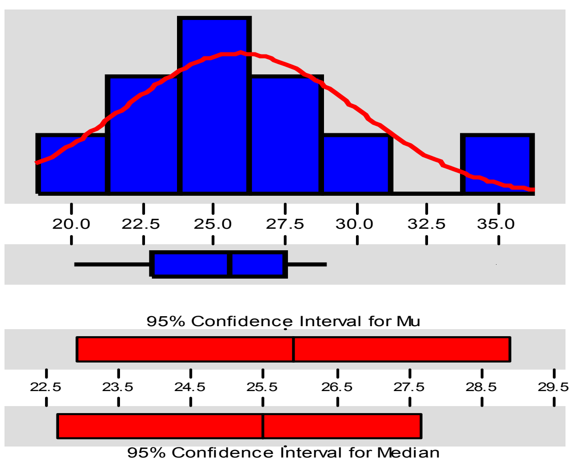
Figure A

Trial	Set 1 (Air-filled football)	Set 2 (Helium-filled football)
2	23	16
7	26	25
13	27	28
18	22	26
24	29	32
29	25	33
31	27	26
36	25	29
17	20	23
5	35	23



The “Descriptive Statistics” below for both air and helium-filled footballs lists some of the basic statistics calculated from the 10 observations. An analysis of the basic statistics found between the two samples, Set 1 and Set 2, can be performed. We can see that the sample mean distance traveled by the helium-filled football at 26.10 yards is greater than the sample mean distance traveled by the air-filled football at 25.90 yards. In looking at the sample standard deviation and the sample variance, we can see that the 10 observations from the helium-filled football had more variation in their distances than the 10 observations from the air-filled football. In other words,
$(S_2^2 = 24.1) > (S_1^2 = 17.2)$.

Descriptive Statistics



Variable: Air S

Anderson-Darling Normality Test

A-Squared:	0.318
P-Value:	0.474
Mean	25.9000
StDev	4.1486
Variance	17.2111
Skewness	0.963785
Kurtosis	1.89978
N	10
Minimum	20.0000
1st Quartile	22.7500
Median	25.5000
3rd Quartile	27.5000
Maximum	35.0000

95% Confidence Interval for Mu

22.9323 28.8677

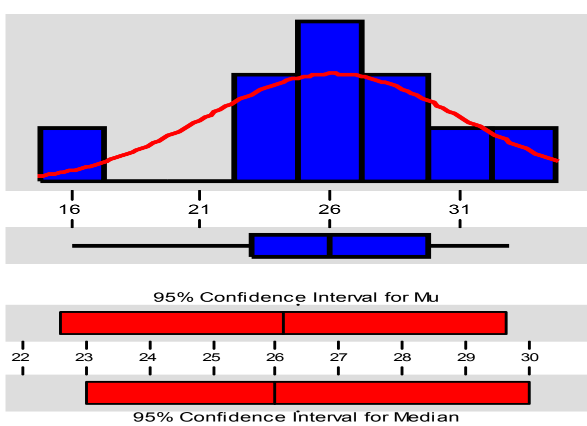
95% Confidence Interval for Sigma

2.8536 7.5738

95% Confidence Interval for Median

22.6577 27.6847

Descriptive Statistics



Variable: Helium S

Anderson-Darling Normality Test

A-Squared:	0.251
P-Value:	0.659
Mean	26.1000
StDev	4.9092
Variance	24.1
Skewness	-6.2E-01
Kurtosis	0.979593
N	10
Minimum	16.0000
1st Quartile	23.0000
Median	26.0000
3rd Quartile	29.7500
Maximum	33.0000

95% Confidence Interval for Mu

22.5882 29.6118

95% Confidence Interval for Sigma

3.3767 8.9622

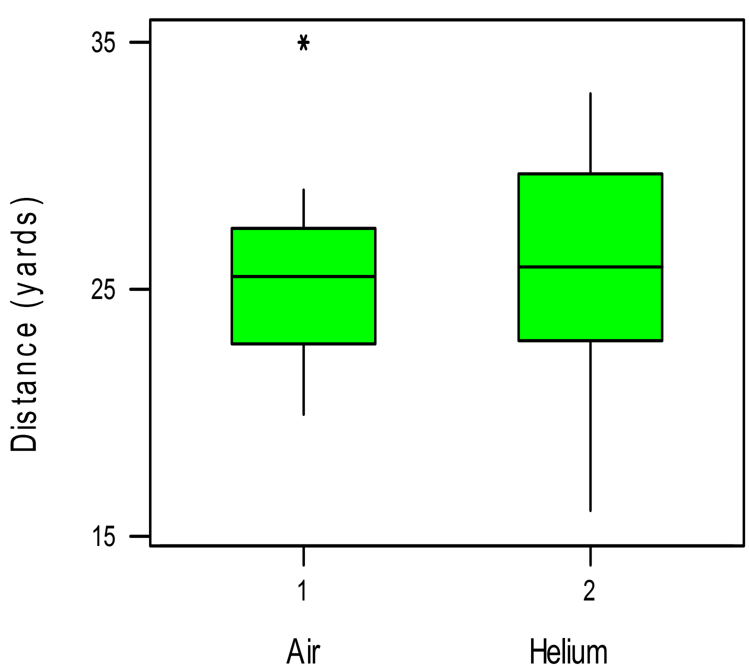
95% Confidence Interval for Median

23.0000 30.0270

<u>**Boxplots**</u>

Some of the other data on the "Descriptive Statistics" pages can be viewed graphically through the boxplots in Figure B where boxplot 1 corresponds to the air-filled football data and boxplot 2 corresponds to the helium-filled football data. The first thing to notice is that the actual "box" of boxplot 2 is larger than that of boxplot 1. This difference can be explained by the differences in the sample variance as noted above. Since $S_2^2 > S_1^2$, Set 2 has more variability in its 10 observations, and therefore the middle 50% of the observations for Set 2 (the "box") will cover more range than the middle 50% of the observations will for Set 1. Another difference to notice is the outlier denoted by the asterisk in boxplot 1. An outlier in Minitab boxplot output represents any observation that is greater than 2 standard deviations away from the mean of the sample. This outlier could be possibly affecting the calculation of sample mean and sample variance for Set 1. A similarity between the boxplots is the placement of the median, the line drawn inside the "box." It is difficult to tell whether there is any skewness within these boxplots. Therefore we will next analyze histograms with normal curves.

Figure B



Histograms

Figure C and Figure D represent histograms with fitted normal curves for the air-filled football data and the helium-filled football data, respectively. The histogram gives us a representation of how often a certain value of an observation occurs during the experiment. One noticeable feature of both histograms is some slight skewness. The histogram of air-filled footballs seems to be slightly skewed to the left, whereas the histogram of helium-filled footballs seems to be slightly skewed to the right. However, this occurrence could be due to the 10 samples that were randomly selected. If we had a choice of the 10 samples we wanted to analyze, we could select a sample that contained distances closer to the sample mean. For example, trial 5 contained a distance of 35 yards for the air-filled football which represents the outlier. We could possibly replace this trial with another and get a histogram that is not skewed. Also, trial 2 contains an observation of only 16 yards for the helium-filled football which disrupts the normality of the histogram. However, even with these two particular observations, it is possible to assume that the distribution of Set 1 and Set 2 are approximately normal.

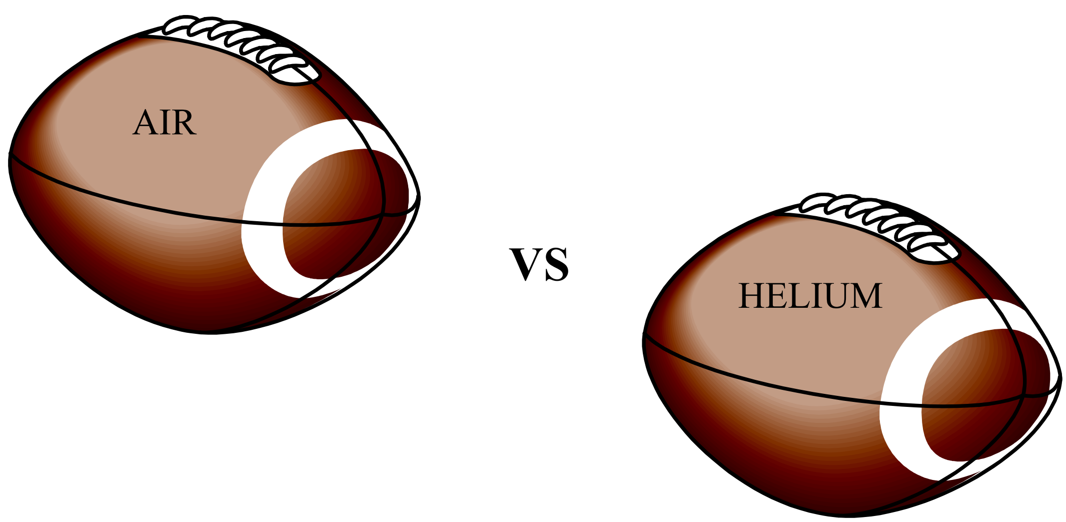


Figure C

Histogram of Air S, with Normal Curve

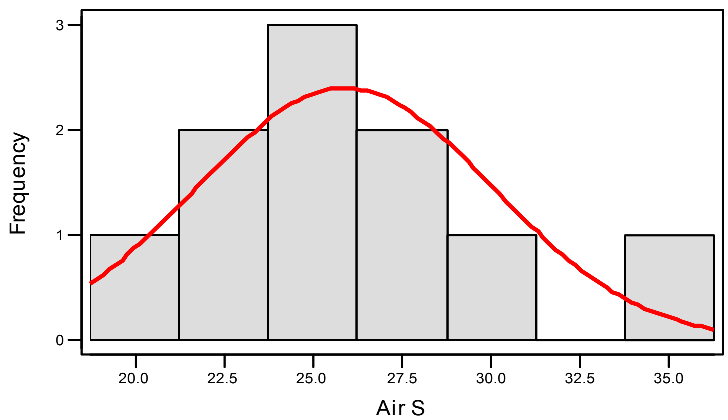
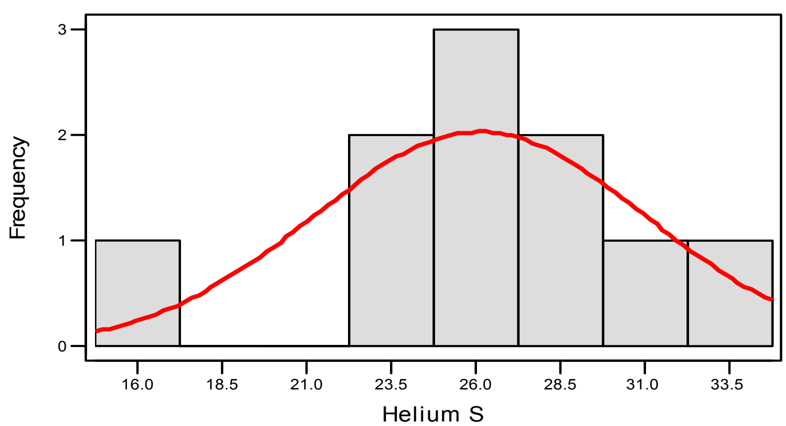


Figure D

Histogram of Helium S, with Normal Curve



Confidence Intervals for the mean of the population (in yards):

Because the sample size (n = 10) is relatively small, we assume that the population is normal and use the t-distribution with n – 1 degrees of freedom.

C.I. for μ: $\bar{X} \pm t_{n-1, \alpha/2} (S/n^{1/2})$

Air:	**Helium:**
90% C.I. for μ : [23.4945, 28.3055]	90% C.I. for μ : [23.2639, 28.9461]
95% C.I. for μ : [22.9315, 28.8685]	95% C.I. for μ : [22.5878, 29.6122]
99% C.I. for μ : [21.6349, 30.1651]	99% C.I. for μ : [21.0538, 31.1462]

Notice that the confidence intervals at every level cover a larger distance for the helium-filled data than the air-filled data. This difference is due to the fact that $S_2^2 > S_1^2$. In other words, data Set 2 with higher variability is required to cover more values in order for its confidence level to be the same as for Set 1 with lower variability. An example of a confidence statement is that we are 95% confident that the true mean of the populations for air-filled footballs is between 22.9315 yards and 28.8685 yards. Also, we are 95% confident that the true mean of the populations for helium-filled footballs is between 22.5878 yards and 29.6122 yards.

Confidence Intervals for the difference of the means of the populations (in yards):

Once again, because n = 10 is a relatively small sample size, we assume independent random samples both from normally distributed populations and use the t-distribution with $n_1 + n_2 - 2$ degrees of freedom. We will also assume that the population variances are equal and use S_p^2, a pooled unbiased estimator for the population variance.

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = 20.6653$$

C.I. for $\mu_1 - \mu_2$: $(\bar{X}_1 - \bar{X}_2) \pm t_{n1 + n2 - 2, \alpha/2} * S_p\{(1/n_1) + (1/n_2)\}^{1/2}$

90% C.I. for $\mu_1 - \mu_2$ = [-3.7252, 3.3252]

95% C.I. for $\mu_1 - \mu_2$ = [-4.4713, 4.0713]

99% C.I. for $\mu_1 - \mu_2$ = [-6.051, 5.651]

An example of a confidence statement is that we are 95% confident that the true mean difference between the two populations is between –4.4713 yards and 4.0713 yards.

Notice that at all confidence levels, the difference $\mu_1 - \mu_2 = 0$ is included. Therefore we cannot rule out the possibility that the true means of the populations are equal.

Confidence Intervals for population variance (in yards):

For these calculations, we once again assume we are sampling from a normal population, and we will need the Chi-square distribution with n – 1 degrees of freedom.

C.I. for σ^2: $\frac{(n-1)S^2}{\chi^2_{\alpha/2,\ n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,\ n-1}}$

Air:	**Helium:**
90% C.I. for σ^2 = [9.1554, 46.5864]	90% C.I. for σ^2 = [12.8199, 65.2331]
95% C.I. for σ^2 = [8.1428, 57.3703]	95% C.I. for σ^2 = [11.402, 80.3333]
99% C.I. for σ^2 = [6.5666, 89.2795]	99% C.I. for σ^2 = [9.195, 125.0144]

An example of a confidence statement is that we are 95% confident that the true population variance for air-filled footballs is between 8.1428 yards and 57.3703 yards. Or, we are 95% confident that the true population variance for helium-filled footballs is between 11.402yards and 80.3333 yards. Notice that at every confidence level, the minimum and maximum confidence limits are higher for Set 2 than for Set 1 explaining the higher variability in Set 2 than in Set 1. Another interesting characteristic is that the confidence intervals at every confidence level cover a larger range of values for Set 2 than for Set 1.

Confidence Intervals for the ratio of the population variances (in yards):

For these calculations, we will assume independent random samples from normal populations and use the f-distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.

C.I. for σ_3^2/σ_4^2: $\frac{S_1^2}{S_2^2} * \frac{1}{f_{\alpha/2,\, n1-1,\, n2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} * f_{\alpha/2,\, n1-1,\, n2-1}$

90% C.I. for σ_1^2/σ_2^2 = [.2246, 2.2710]

95% C.I. for σ_1^2/σ_2^2 = [.1772, 2.8780]

99% C.I. for σ_1^2/σ_2^2 = [.1092, 4.6706]

An example of a confidence statement is that we are 95% confident that the true ratio of the variances is between .1772 yards and 2.8780 yards. Notice that at every confidence level, $\sigma_1^2/\sigma_2^2 = 1$ is included. Therefore, we cannot rule out the possibility that the true population variances could be equal.

Conclusion

What can we conclude from all of this analysis? By directly comparing the sample means, we saw that the mean distance traveled by the helium-filled football was greater than the mean distance traveled by the air-filled football. This might lead us to suggest that the helium-filled football on average travels farther than the air-filled football. By comparing the sample variances, we saw that the variability in the distance traveled by the helium-filled football was greater than the air-filled football suggesting that the air-filled football was more consistent. However, we are only sampling 10 trials here with not much information. Therefore, we assumed an approximation to a normal distribution and calculated confidence intervals. Two major characteristics of the confidence intervals concerning the means should be mentioned. First, notice that the confidence intervals for the population means of air-filled and helium-filled footballs all have relatively similar confidence limits. Secondly, notice that the confidence intervals for the difference between the population means at every significance level include the value "0." These two characteristics imply that we cannot be very confident in saying that for example helium-filled footballs on average travel farther than air-filled footballs. There are also two major characteristics of the confidence intervals concerning the population variances. First, notice that the confidence intervals for population variance of helium-filled footballs all have higher confidence limits than those for air-filled footballs. Secondly, notice that the confidence intervals for the ratio of the population variance at every significance level include the value "1." The first characteristic implies that the helium-football does indeed have more variability, or that it is not as consistent as the air-filled football. However, the second characteristic says that we cannot be very confident that the population variances are not equal.

We can see that there are obvious differences between the two data sets. The statistics that we calculated for sample mean and sample variance differed between the two data sets, and the data sets resulted in boxplots and histograms that didn't seem to exactly match up. However, after analyzing the confidence intervals, we cannot confidently say that there is a large difference in the mean distance traveled by each football, nor can we confidently say that there is a large difference in the variability of the footballs.