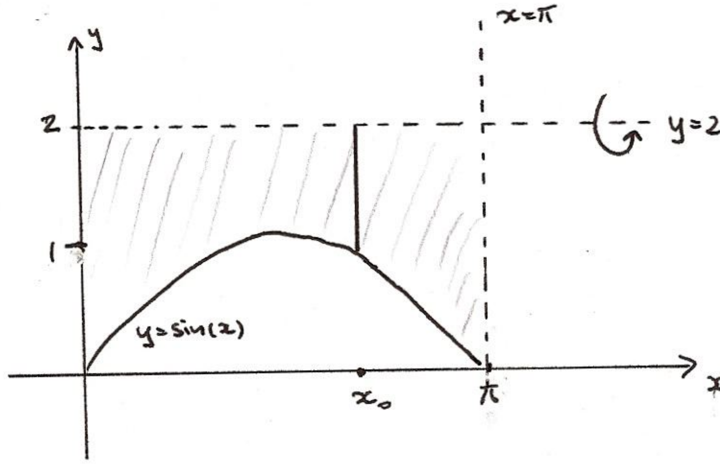
PROBLEM 2* Find the volume of the solid generated by revolving the region bounded by the curve $y = \sin(x)$ and the lines $x = 0$, $x = \pi$, and $y = 2$ about the line $y = 2$.




The solid is positioned between planes $x=0$ and $x=\pi$. The cross-section of the solid by a plane

$$x = x_0$$

(where $x_0 \in [0,\pi]$) is a disc with radius running from the point $(x_0, 2, 0)$ to the point $(x_0, \sin(x_0), 0)$. Thus the length of the radius of this disc is $2-\sin(x_0)$. Consequently, the area of the cross-section at $x_0$ is

$$A(x_0) = \pi \cdot (2-\sin(x_0))^2 = 4\pi - 4\pi\sin(x_0) + \pi\sin^2(x_0)$$

The function $A(x)$, $0 \le x \le \pi$, is continuous (so integrable). Therefore we may use the definition of the volume of a solid to claim that

$$V = \int_0^\pi A(x)\,dx = \int_0^\pi 4\pi - 4\pi\sin(x) + \pi\sin^2(x)\,dx =$$

$$= 4\pi x\Big]_0^\pi + 4\pi\cos(x)\Big]_0^\pi + \left(\tfrac{\pi}{2}x - \tfrac{\pi}{4}\sin(2x)\right)\Big]_0^\pi =$$

$$= 4\pi^2 - 4\pi - 4\pi + \frac{\pi^2}{2} = \frac{9}{2}\pi^2 - 8\pi$$

Thus the volume of the solid in question is

$$V = \frac{9}{2}\pi^2 - 8\pi.$$